Diffractive neutrino-production of pions in the color dipole model

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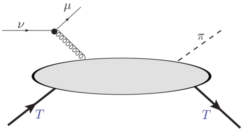
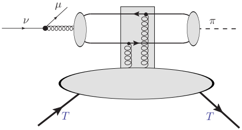
Outline

Kinemaitcs



- Diffractive pion production, $\nu T \to \mu \pi T$
  - T is either proton or nucleus
  - neutrino may be ν_μ, ν_e

Kinemaitcs



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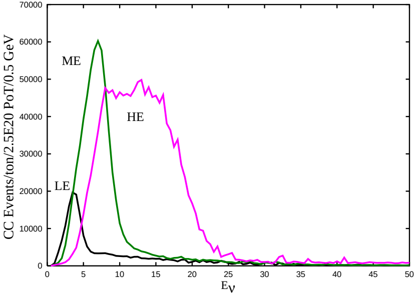
- Diffractive kinematics, energy $\nu \gg \nu_{min} \sim m_\pi^2 R_A$
  Minerva Proposal, 2004:



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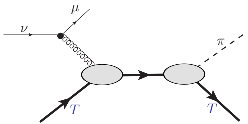

- Diffractive pion production, $\nu T \to \mu \pi T$

- Diffractive kinematics, energy $\nu \gg \nu_{min} \sim m_\pi^2 R_A$
- Not valid for the small-ν region where dominant contribution comes from resonances

Adler theorem & PCAC

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$$L_{\mu\nu} = 2\frac{E_\nu (E_\nu - \nu)}{\nu^2} q_\mu q_\nu + \mathscr{O}\left(q^2\right) + \mathscr{O}\left(m_l^2\right)$$

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- So the cross-section may be evaluated using the PCAC hypothesis (*S. Adler, 1966*)

$$\left.\frac{d\sigma_{\nu T \to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T \to F}$$

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- In addition, we have contributions from transverse part and from the vector part ($\mathscr{O}\left(q^2\right)$ for small q^2)

Black disk limit

Adler relation is inconsistent with black disk limit: consider single-pion production,

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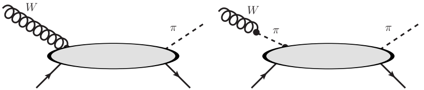

$$\sim \frac{q_\mu}{q^2-m_\pi}$$

(pions do not contribute due to lepton current conservation)

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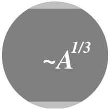
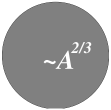


$\sim 2\pi R$

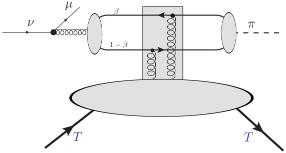


$\sim \pi R^2$

Energy dependence in Froissart limit:

$\sim \ln s$ $\qquad$ $\sim \ln^2 s$

Color dipole and neutrino-proton interactions



The amplitude has a form

$$\mathscr{A}^{aT\to\pi T}=\int d\beta\, d\beta' d^2 r d^2 r' \bar{\Psi}_\pi\left(\beta',r'\right)\mathscr{A}_T^d\left(\beta',r';\beta,r\right)\Psi_a\left(\beta,r\right),$$

- $\mathscr{A}_T^d\left(\beta',r';\beta,r\right)$ universal object, depends only on the target T. Known from photon-proton and photon-nuclear processes
- $\bar{\Psi}_\pi,\Psi_a$ are the distribution amplitudes of the initial and final states

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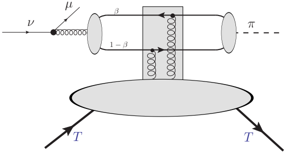

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- $\bar{\Psi}_\pi,\Psi_a$ are the distribution amplitudes of the initial and final states
- Earlier applications of color dipole model:
  - Formulated for photon-proton and proton-nuclear processes (vector current)
  - Applications to processes with neutrinos (vector current)
    - electroweak DVCS (*Machado 2007*)
    - electroweak DIS (*Fiore, Zoller 2005; Gay Ducati, Machado 2007*)
    - charm/heavy meson production (*Fiore, Zoller 2009; Gay Ducati, Machado 2009*)

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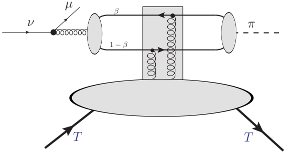

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    - charm/heavy meson production (*Fiore, Zoller 2009; Gay Ducati, Machado 2009*)
- We are going to use color dipole for description of the axial current

Extension from vector to axial current

Extension of effective models from vector to axial current is not straightforward.
Example: extension of Generalized Vector meson Dominance (GVMD) leads to Piketty-Stodolsky paradox:

$$\sigma_{\pi p \to \pi p} \neq \sigma_{\pi p \to a_1 p}$$

- VMD does not work for axial current, dominant contributions comes from multimeson states $(\rho\pi, \pi\pi\pi, \ldots)$ (*Belkov, Kopeliovich, 1986*)

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- In color dipole we do not have such problems since there is no explicit hadrons like in GVMD

Distribution amplitudes from the Instanton Vacuum Model

Why IVM ?

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- The model is valid for low virtualities Q^2
- The model has built-in chiral symmetry
- Effective action :

$$S=\int d^4x\left(2\Phi^\dagger(x)\Phi(x)-\bar{\psi}\left(\hat{p}+\hat{v}+\hat{a}\gamma_5-m-c\bar{L}f\otimes\Phi\cdot\Gamma_m\otimes fL\right)\psi\right),$$

  - has only two parameters (average instanton size $\rho\sim 1/600 MeV$ and average distance $R\sim 1/200 MeV$), but reproduces the low-energy constants in chiral lagrangian.
  - may be rewritten as NJL with nonlocal interactions (nonlocality from instanton shape)

Distribution amplitudes of pion

Pion distribution amplitudes (P. Ball *et al*, 2006)

$$\begin{aligned}\left\langle 0\left|\bar{\psi}(y)\gamma_\mu\gamma_5\psi(x)\right|\pi(q)\right\rangle &= if_\pi\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\times \\ &\times \left(p_\mu\phi_{2;\pi}(u)+\frac{1}{2}\frac{z_\mu}{(p\cdot z)}\psi_{4;\pi}(u)\right),\end{aligned}$$

$$\langle 0|\bar{\psi}(y)\gamma_5\psi(x)|\pi(q)\rangle = -if_\pi\frac{m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\phi_{3;\pi}^{(p)}(u).$$

$$\begin{aligned}\left\langle 0\left|\bar{\psi}(y)\sigma_{\mu\nu}\gamma_5\psi(x)\right|\pi(q)\right\rangle &= -\frac{i}{3}f_\pi\frac{m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\times \\ &\times \frac{1}{p\cdot z}\left(p_\mu z_\nu-p_\nu z_\mu\right)\phi_{3;\pi}^{(\sigma)}(u),\end{aligned}$$

Distribution amplitudes of axial meson

Axial distribution amplitudes (K.-C. Yang 2007)

$$\begin{aligned}\left\langle 0\left|\bar{\psi}(y)\gamma_\mu\gamma_5\psi(x)\right|A(q)\right\rangle &= if_A m_A\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\times\\ &\times\left(p_\mu\frac{e^{(\lambda)}\cdot z}{p\cdot z}\Phi_{||}(u)+e_\mu^{(\lambda=\perp)}g_\perp^{(a)}(u)-\frac{1}{2}z_\mu\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_A^2 g_3(u)\right),\end{aligned}$$

$$\left\langle 0\left|\bar{\psi}(y)\gamma_\mu\psi(x)\right|A(q)\right\rangle = -if_A m_A\varepsilon_{\mu\nu\rho\sigma}e_\nu^{(\lambda)}p_\rho z_\sigma\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\frac{g_\perp^{(v)}(u)}{4}$$

$$\begin{aligned}\left\langle 0\left|\bar{\psi}(y)\sigma_{\mu\nu}\gamma_5\psi(x)\right|A(q)\right\rangle &= f_A^\perp\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\left(e_{[\mu}^{(\lambda=\perp)}p_{\nu]}\Phi_\perp(u)\right.\\ &+\left.\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_A^2 p_{[\mu}z_{\nu]}h_{||}^{(t)}(u)+\frac{1}{2}e_{[\mu}^{(\lambda)}z_{\nu]}\frac{m_A^2}{p\cdot z}h_3(u)\right),\end{aligned}$$

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Color dipole in the black disk regime

In the black disk limit all partial amplitudes reach unitarity bound, $a_l \to 1$. Respectively color dipole amplitude $\mathscr{A}^d(r) \approx const$, so the result is proportional to the convolution

$$\begin{aligned}\langle\bar{\Phi}_a \Phi_\pi\rangle &= \int d\beta\, d^2 r\, \bar{\Psi}_\pi(\beta, r)\, \Psi_a(\beta, r) \\ &\sim m_\pi^2 \frac{q_\mu}{q^2}\left(1 + \mathscr{O}\left(\frac{m_\pi^2}{q^2}\right)\right);\end{aligned}$$

after convolution with lepton current l_μ and due to conservation of lepton vector/axial currents (assume $m_l \to 0$) we get exactly zero.

Coherent neutrino-nuclear scattering

- Two different coherence lengths: coherence length of the pion

$$l_c^\pi = \frac{2\nu}{m_\pi^2 + Q^2}$$

and coherence length of the effective axial meson state,

$$l_c^a = \frac{2\nu}{m_a^2 + Q^2}.$$

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- For large Q^2, $l_c^\pi \approx l_c^a$, so this case is similar to photon-nuclear processes, we have only two regimes: $l_c \gg R_A$ and $l_c \ll R_A$.
- For small $m_\pi^2 \lesssim Q^2 \ll m_a^2$ the two scales are essentially different, $l_c^a \ll l_c^\pi$, so there are three regimes depending on relations between R_A and l_c^a, l_c^π.

Coherent neutrino-nuclear scattering (contd.)

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- $R_A \ll l_c^a \ll l_c^\pi$: absorptive corrections are large, Adler relation is not valid even for $Q^2 = 0$, $\sigma \sim A^{1/3}$

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

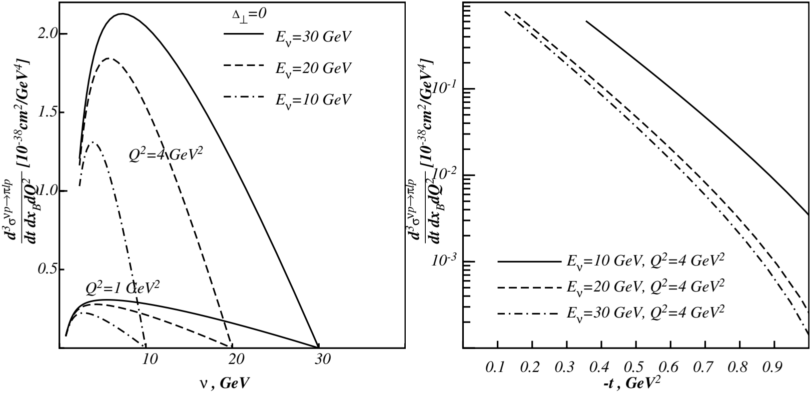

Figure: Differential cross-section $d\sigma/d\nu dt dQ^2$ for different neutrino energies E_ν.

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

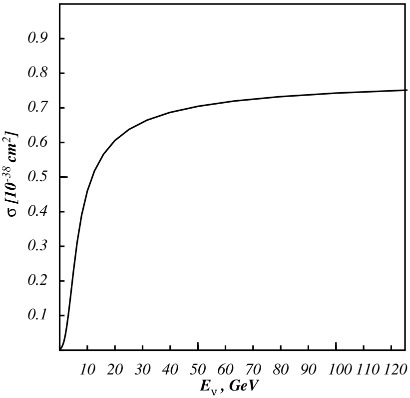

Figure: Total cross-section as a function of the neutrino energy E_ν.

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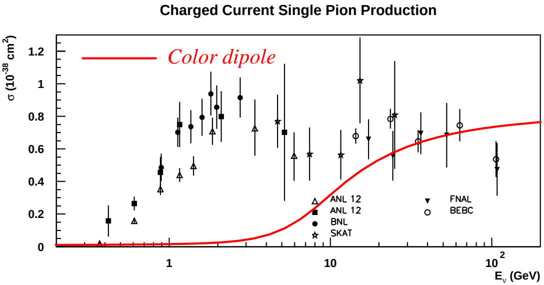
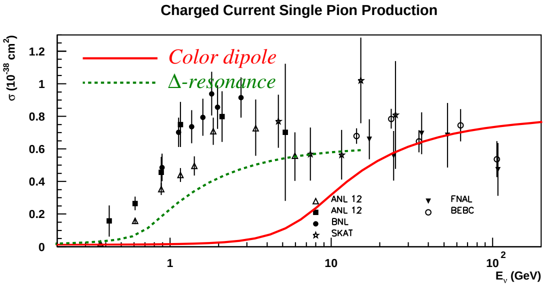


Figure: Total cross-section as a function of the neutrino energy E_ν. *Compilation of experimental data from Minerva proposal, 2004*

Agreement for energies $E_\nu > 10$ GeV, problem for $E_\nu < 10$ GeV

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section



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Low-energy region is dominated by Δ

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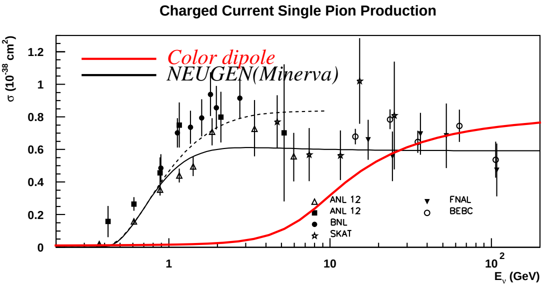

Figure: Total cross-section as a function of the neutrino energy E_ν. *Compilation of experimental data from Minerva proposal, 2004*

Difference between NEUGEN and color dipole: cross-section is slowly growing for high energies

Result for the $\nu A \to l\pi^+ A$ differential cross-section

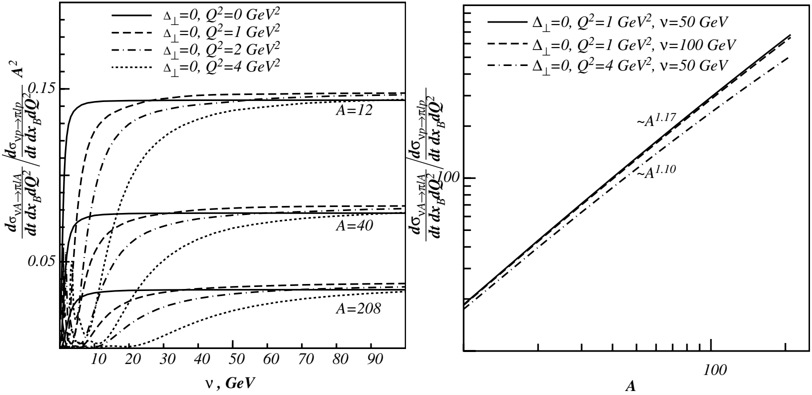

Figure: Ratio of cross-sections on the nucleus and proton.

Conclusion

- We have shown that the Adler relation cannot always be correct for the neutrino-nuclear processes–broken in black disk limit, by absorptive corrections.

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- We have shown that the Adler relation cannot always be correct for the neutrino-nuclear processes–broken in black disk limit, by absorptive corrections.
- We evaluated the results in color dipole model; for small-Q^2 and moderate energies we reproduce Adler theorem; our results are valid also for $Q^2 \neq 0$ (and $\nu \gg m_N$)

Absorptive corrections

- For elastic meson scatterig they have a form

$$\sigma_{el}^{\pi A} \sim \int d^2 b \left(1 - \exp\left(-\frac{1}{2}\sigma_{el}^{\pi N} T_A(b)\right)\right)$$

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- For diffractive meson production they have a form

$$\sigma_{\pi A \to MA} \sim \int d^2 b \frac{\exp\left(-\frac{1}{2}\sigma_{el}^{\pi N} T_A(b)\right) - \exp\left(-\frac{1}{2}\sigma_{el}^{MN} T_A(b)\right)}{\sigma_{el}^{\pi N} - \sigma_{el}^{MN}} \approx$$

$$\approx \int d^2 b \exp\left(-\frac{1}{2}\sigma_{el}^{\pi N} T_A(b)\right)$$

-different in black disk limit

PCAC vs. pion dominance

Adler theorem: replace W with π for $Q^2 = 0$

$$\left.\frac{d\sigma_{\nu T\to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T\to F}$$

Pion dominance model:

$$T_\mu(...) \sim \frac{q_\mu}{q^2 - m_\pi^2} + T_\mu^{non-pion}(...),$$

but lepton currents are conserved, so

$$q_\mu L_{\mu\nu} = \mathcal{O}(m_l)$$

$\Rightarrow$contribution of pions is zero$\Rightarrow$contribution of non-pions should exactly match to the contribution of pions

Chiral symmetry & PCAC

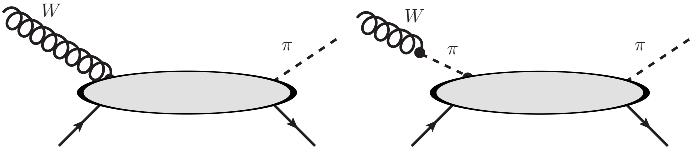

Figure: W may couple directly to quarks in the target or via intermediate pion

$$\mathscr{L}_2 \approx \frac{F^2}{2}\left(\partial_\mu \vec{\phi} - \vec{a}_\mu\right)^2 + \mathscr{O}\left(m, \phi^3, a^3, a^2\phi, \dots\right),$$

$$\mathscr{L}_{\pi N}^{(1)} \approx \bar{\Psi}\left(i\gamma_\mu\partial_\mu + m_N - i\frac{g_A}{4}\gamma_\mu\gamma_5\left(\vec{a}_\mu - \partial_\mu\vec{\phi}\right)\right)\Psi + \mathscr{O}\left(m, \phi^3, a^3, a^2\phi, \dots\right).$$

$$T_\mu^{(a\to\pi)} = T_{\pi\pi}(p,q)\left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right)P_\nu\left(p, \Delta\right),$$

Chiral symmetry & PCAC & color dipole

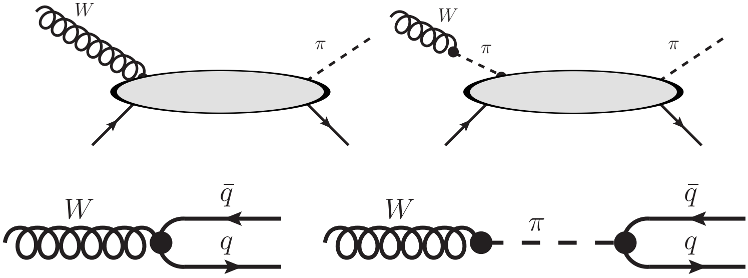

Figure: Relation between couplings $\pi\bar{q}q$, $W\bar{q}q, W\pi$ guraantees that the amplitude remains transverse

$$T_{\mu}^{(a\to\pi)}=T_{\pi\pi}(p,q)\left(\frac{q_{\mu}q_{\nu}}{q^{2}-m_{\pi}^{2}}-g_{\mu\nu}\right)P_{\nu}\left(p,\Delta\right),$$